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SOLUTIONS OF PROBLEMS.

ALGEBRA.

459. Proposed by C. N. SCHMALL, New York City.

By d'Alembert's test, or otherwise, show that in the infinite series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \cdots + \frac{n^n x^n}{n!} + \cdots$$

the upper limit of the interval of convergence is $1/e$, where e is the Napierian base, *i. e.*, $x < 1/e$ when the series is convergent. (Bromwich's *Infinite Series*, pp. 28, 33.)

SOLUTION BY HORACE OLSON, Chicago, Illinois.

Let r_n represent the ratio of the $(n+1)$ th term of the given series to the n th term. Then

$$\lim_{n \rightarrow \infty} r_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n x = ex.$$

Therefore, the series is convergent if $x < 1/e$, and divergent if $x > 1/e$.

$$\text{If } x = 1/e, r_n = \left(1 + \frac{1}{n}\right)^n \cdot \frac{1}{e}.$$

Let ρ_n be the ratio of the $(n+1)$ th term of the harmonic series to the n th term; *i. e.*, $\rho_n = n/(n+1)$. Then $r_n/\rho_n = 1/[1 + (1/n)]^{n+1}$.

$[1 + (1/n)]^{n+1}$ is a decreasing function of n , since its derivative

$$\left(1 + \frac{1}{n}\right)^{n+1} \left\{ \log \left(1 + \frac{1}{n}\right) - \frac{1}{n} \right\} < 0.$$

But

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n+1} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

Therefore r_n/ρ_n is always greater than unity.

Therefore the n th term of the given series is always greater than $1/e$ times the n th term of the harmonic series, and, since the latter is divergent, the former also is divergent.

Therefore, finally, the given series is convergent for $x < 1/e$ and divergent for $x \geq 1/e$.

Also solved by E. W. WORTHINGTON, A. M. HARDING, G. W. HARTWELL, E. J. OGLESBY, and the PROPOSER.

460. Proposed by J. J. GINSBURG, Student, Cooper Union, New York.

Find the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}$ to infinity.

SOLUTION BY NATHAN ALTSHILLER, University of Oklahoma.

The number of operations increasing indefinitely, the above expression has no value, strictly speaking, unless the word "value" is taken in this connection to be the exact equivalent of the word "limit." The problem means: given a sequence of terms

$$u_1 = \sqrt{1}, \quad u_2 = \sqrt{1 + \sqrt{1}}, \quad u_3 = \sqrt{1 + \sqrt{1 + \sqrt{1}}}, \quad \cdots \quad u_n = \cdots$$

find the limit of u_n when n increases indefinitely, if such a limit exists.

The problem may be generalized by considering $\sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a + \cdots}}}}$ to infinity, where a is any positive quantity.

We observe that the values of the u 's increase with n so that (1) $u_{n+1} > u_n$. It is also easily seen that (2) $u_{n+1}^2 = u_n + a$.

From (1) and (2) it follows that $u_n^2 - u_n - a < 0$.